PHYS 100C Midterm, Tuesday May 7, 11:00-12:20 (1hr 20min)

1. An electromagnetic wave is described by the equations: ignore hats in the exponents

$$\boldsymbol{E}(x,y,z,t) = E_0 e^{ik\left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right)-i\omega t}\,\hat{\boldsymbol{z}}$$

$$\boldsymbol{B}(x,y,z,t) = B_0 e^{ik\left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right)-i\omega t}\,\frac{\hat{\boldsymbol{x}}-\hat{\boldsymbol{y}}}{\sqrt{2}}$$

   a) Describe the direction of wave propagation
   b) This wave is incident on a mirror-like planar perfectly conducting surface described by the equation x=0. Write down the equations for the E and B components of the reflected wave.

2. Find the radiation pressure resulting from sunlight with ~~power~~ intensity 1kW/m^2 normally incident at an air-glass interface with index of refraction n=1.5

3. Waves on a guitar string have the dispersion relation:

$$\omega^2 = \frac{T}{\mu}k^2 + \alpha\, k^4$$

   (here T is the string tension, μ is the string mass per unit length and α is a constant).
   a) Find the expression for the ratio of group and phase velocities of the guitar string waves
   b) What is the value for this ratio in the limit of very small k? In the large k limit?

4. Transform the potentials for an infinite charged solenoid given below (in cylindrical coordinates, with ρ, n, I and R as constants) so that the new potential $V'(r,\varphi)$=0 and find the corresponding $A'(r,\varphi)$ (up to a constant).

$$V(r,\varphi) = \frac{\rho}{2\pi\varepsilon_0 r}\ , \quad \boldsymbol{A}(r,\varphi) = \frac{\mu_0 nI}{2}\frac{R^2}{r}\hat{\boldsymbol{\varphi}}$$

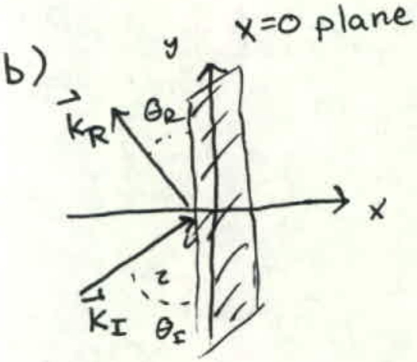
① a) $\vec{E}_I = E_0 e^{ik\left(\frac{x+y}{\sqrt{2}}\right) - i\omega t}\,\hat{z}$ $\qquad \hat{n} = \hat{z}$

$\vec{B}_I = B_0 e^{ik\left(\frac{x+y}{\sqrt{2}}\right) - i\omega t} \cdot \frac{\hat{x}-\hat{y}}{\sqrt{2}}$ $\qquad \hat{k}\times\hat{n} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{vmatrix} = \frac{\hat{x}}{\sqrt{2}} - \frac{\hat{y}}{\sqrt{2}}$

Describe the direction of wave propagation: $\vec{k}\cdot\vec{r} = k\,\frac{x+y}{\sqrt{2}}$

direction $\hat{k} = \frac{\hat{x}+\hat{y}}{\sqrt{2}} \rightarrow$ $\boxed{\text{upwards at a } 45^\circ \text{ angle in the xy-plane}}$ 3

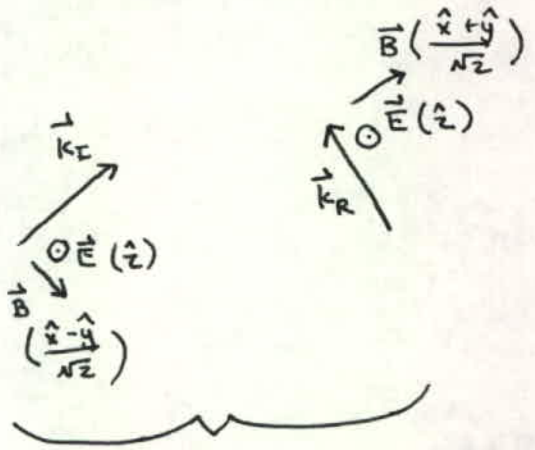
b)



perfectly conducting surface, 100% reflecting so there is no transmitted wave

$\theta_I = \theta_R$ law of reflection 1

at a conducting surface, there is a 180° phase shift to the $\vec{E}$ and $\vec{B}$ fields to satisfy the boundary conditions 2

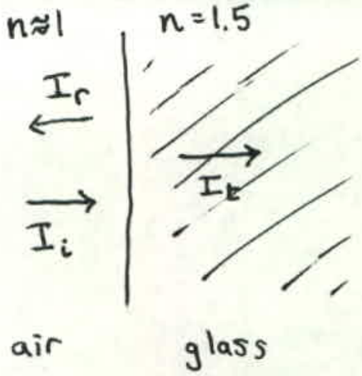


$\vec{E}\times\vec{B}$ must give direction of propagation (Poynting vector)

$$\boxed{\begin{aligned} \vec{E}_R &= E_0 e^{ik\left(\frac{-x+y}{\sqrt{2}}\right) - i\omega t + \pi}\,\hat{z} \\ &= -E_0 e^{ik\left(\frac{-x+y}{\sqrt{2}}\right) - i\omega t}\,\hat{z} \\ \vec{B}_R &= B_0 e^{ik\left(\frac{-x+y}{\sqrt{2}}\right) - i\omega t + \pi} \cdot \frac{\hat{x}+\hat{y}}{\sqrt{2}} \\ &= -B_0 e^{ik\left(\frac{-x+y}{\sqrt{2}}\right) - i\omega t} \cdot \frac{\hat{x}+\hat{y}}{\sqrt{2}} \end{aligned}}$$ 2 2

wave starts travelling in −x direction

②



$I_i = 1 kW/m^2$

reflection and transmission at normal incidence

Fresnel Equations →

$$R \equiv \frac{I_R}{I_i} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1.5 - 1}{1.5 + 1}\right)^2 = 0.04$$

$$T \equiv \frac{I_T}{I_i} = \frac{4 n_1 n_2}{(n_1 + n_2)^2} = \frac{4(1.5)(1)}{(1.5+1)^2} = 0.96$$

radiation pressure $P = \frac{I}{c}$ (absorbed / transmited)

$$P = \frac{2I}{c} \quad \text{(reflected)}$$

$$P_{total} = \frac{(0.96)(1\ kW/m^2) + 2(0.04)(1\ kW/m^2)}{3 \cdot 10^8\ m/s} = \boxed{\cdot 10^{-6}\ N/m^2}$$

3.47

③ a) $\omega^2 = \frac{T}{\mu} k^2 + \alpha k^4 \Rightarrow \omega = \left(\frac{T}{\mu} k^2 + \alpha k^4\right)^{1/2}$

$$v_g = \frac{d\omega}{dk} = \frac{1}{\cancel{2}}\left(\frac{T}{\mu}k^2 + \alpha k^4\right)^{-1/2} \cdot \left(\cancel{2}\frac{T}{\mu}k + \overset{2}{\cancel{4}}\alpha k^3\right)$$

$$v_p = \frac{\omega}{k} = \frac{1}{k}\left(\frac{T}{\mu}k^2 + \alpha k^4\right)^{+1/2}$$

$$\frac{v_g}{v_p} = \frac{k \cdot \left(\frac{T}{\mu}k + 2\alpha k^3\right)}{\left(\frac{T}{\mu}k^2 + \alpha k^4\right)^{1/2}\left(\frac{T}{\mu}k^2 + \alpha k^4\right)^{1/2}} = \boxed{\frac{\frac{T}{\mu}k^2 + 2\alpha k^4}{\frac{T}{\mu}k^2 + \alpha k^4}}$$

b) in the limit $k \to 0$, the k^2 term dominates so $\frac{v_g}{v_p} \to \boxed{1}$

" " " $k \to \infty$, " k^4 " " so $\frac{v_g}{v_p} \to \frac{2\alpha k^4}{k^4} \to \boxed{2}$

④ $V(r,\varphi) = \frac{\rho}{2\pi\epsilon_0 r}$ $\vec{A}(r,\varphi) = \frac{\mu_0 n I}{2} \frac{R^2}{r} \hat{\varphi}$

gauge transformation $\begin{cases} V' = V - \frac{\partial\lambda}{\partial t} \\ \vec{A}' = \vec{A} + \nabla\lambda \end{cases}$

if $V' = 0$, this means $\int \frac{\partial\lambda}{\partial t} dt = \int \frac{\rho}{2\pi\epsilon_0 r} dt$

$$\lambda = \frac{\rho t}{2\pi\epsilon_0 r} + C(r,\theta,\varphi)$$

found λ up to a constant
ignore for the rest of the problem

$$\nabla\lambda = \frac{\partial\lambda}{\partial r}\hat{r} + \cancel{\frac{1}{r}\frac{\partial\lambda}{\partial\varphi}}^{0}\hat{\varphi} + \cancel{\frac{\partial\lambda}{\partial z}}^{0}\hat{z}$$

$$\nabla\lambda = \frac{-\rho t}{2\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{A}' = \vec{A} + \nabla\lambda = \boxed{\frac{-\rho t}{2\pi\epsilon_0 r^2}\hat{r} + \frac{\mu_0 n I R^2}{2r}\hat{\varphi}}$$